SET - 2 | GR 14 | CODE: GR14A1002

I B. Tech I Semester Supplementary Examinations, June, 2015
ADVANCED CALCULUS
(Common to CE, EEE, ME, ECE, CSE, BME and IT)

Time: 3 hours **Max Marks: 70**

PART – A
Answer ALL questions
All questions carry equal marks
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10 * 2 Marks = 20 Marks

1). a Given the transformations $u = \frac{x}{y}$, $v = \frac{y}{x}$, show that u and v are functionally dependent. Find the relation. **[2]**

b Find the critical points of the function $f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4$. **[2]**

c Find the length of the curve $x = t$, $y = t^2$ from $t = 0 \to 1$. **[2]**

d The region enclosed by $x = 0,\ y = 0\ and\ x^2 + y^2 = 4$ in the first quadrant revolves about the line $y = 0$. Find the volume of the solid generated. **[2]**

e Evaluate the double integral $\int_0^{\pi}\int_0^{1} \frac{r\ \cos\theta}{1 + r^2}\ dr\ d\theta$. **[2]**

f Given $F(x, y, z) = 4x + 5y + 3z$, evaluate $\iiint_V \left(\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z} \right) dV$, where V is the Cubical solid enclosed by $0 \le x, y, z \le 1$. **[2]**

g Show that Gradient Fields are Irrotational. **[2]**

h Find the divergence and curve for the vector field $\vec{F} = \left(x^2 - y\right)i + 4z\ j + x^2\ k$. **[2]**

i Evaluate by Stoke's theorem $\oint_C \vec{F} . d\vec{r}$ for the vector field $F = \left(x^2 - yz\right)i + \left(y^2 - zx\right)\ j + \left(z^2 - xy\right)k$ and C is the closed curve bounded by the open hemispherical surface $x^2 + y^2 + z^2 = 16$. **[2]**

j Evaluate by Green's Theorem $\oint_C (2x + y)\,dx + (4x + 3y)\ dy$ where C consists of the boundary of the square $0 \le x, y \le 2$, integration being carried out counter clockwise. **[2]**

PART – B

Answer any FIVE questions. All questions carry equal marks

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5 * 10 Marks = 50 Marks

2. Explain how the Hessian matrix is used as a tool in function optimization. Find the absolute maximum or minimum of the function $f(x,y) = x^2 + xy + y^2 - 3x + 3y$ over the triangular region in the first quadrant bounded by the line $x + y = 4$. **[10]**

3. (a) Change the order of integration in the double integral $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\, dx\, dy$ **[10]**

(b) Evaluate the double integral $\iint_R \cos\left(x^2 + y^2\right) dx\, dy$ by transforming in to plane polar coordinates, where R is the region: $x^2 + y^2 \le \frac{\pi}{2},\ y \ge 0$.

4. Draw a rough sketch of the curve $9ay^2 = x(3a - x)^2$. Find the surface area of revolution of the surface generated by the revolution of this curve about the axis of symmetry. **[10]**

5. (a) Find the directional derivative of $Div\,\vec{F}$ at the point $(2,2,1)$ given the field $\vec{F} = x^5 i + y^5 j + z^5 k$ and along the outer normal to the surface $x^2 + y^2 + z^2 = 9$. **[10]**

(b) Evaluate the line integral $\int_C xy\,dx + (x - z)\,dy + xyz\,dz$ where C consists of the parabolic arc $y = x^2$ in $z = 2$ from $(0,0,2) \to (1,1,2)$.

6. Verify the Gauss divergence theorem for the vector field $F = (x + y^2)i + (y + z^2)j + (z + x^2)k$ and S is the closed surface of the cuboid: $0 \le x \le 4, 0 \le y \le 3,\ 0 \le z \le 1$. **[10]**

7. (a) Evaluate the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ given the transformations $u = \frac{yz}{x},\ v = \frac{zx}{y},\ w = \frac{xy}{z}$. [4] **[10]**

(b) Find the volume of the solid obtained by revolving the hyper cycloid $x^{2/3} + y^{2/3} = a^{2/3}$ about any axis of symmetry. [6]

8. Show that the integral $\int_{(0,\,0,\,1)}^{(1,\,\frac{\pi}{2},\,2)} 2xyz^2\, dx + \left(x^2 z^2 + z\cos yz\right)dy + \left(2x^2 yz + y\cos yz\right) dz$ is independent of the path of integration. Evaluate the integral. **[10]**

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